

# Mathematical Models for Estimating Stem Volume and Volume Tables of Rubber Tree

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## Introduction

Rubber tree (*Hevea brasiliensis* Muell Arg.) is an indigenous to the natural forests of Amazon valley in South America. A few seedlings of this species were first introduced in Bangladesh in 1910 to the tea gardens of Baromashia under Fatikchari upzilla in Chittagong district and Amu of Sylhet district from Calcutta Botanical Garden (Ali 1985). Experimental plantations of the species were done in 1954 by Forest Department at Madhupur of Tangail, Hazarikheel of Chittagong and Tetulia of Panchagarh districts. In 1960, commercial plantations were first done by Forest Department in four hectares land at Raojan of Chittagong and 12 ha at Ramu of Cox's Bazar districts. In 1962 commercial plantations of rubber was handed over to Bangladesh Forest Industries Development Corporation (BFIDC) which then made further plantations in Chittagong, Sylhet, Tangail and Sherpur regions of Bangladesh (BFIDC 1995). During the period between 1962 and 1997, Chittagong Hill Tracts Development Board (CHTDB), several multi-national companies and private entrepreneurs raised rubber plantations. Until 2005, all these organizations raised about 27,386 ha of rubber plantations in the country (Sarkar 2006).

Rubber plantations initially was done mainly for latex production and till today all efforts given to increase latex yield. In most cases both government and private owners are in loss selling produced latex only (Arokiaraj *et al.* 2002). Thus, use of rubber wood in making furniture and other end products in addition to latex production would be profitable (Tissari 2002). In Malaysia, rubber wood is extensively used in wood processing industries. The utilization of rubber wood as furniture, pulp, plywood, decorative veneer and novelty items was initiated by Forest Research Institute Malaysia in 1953 (Peel 1958, Peel and Peh 1960); and export of furniture made from rubber wood doubled in 2008 compared to 1998 (Arokiaraj *et al.* 2002). Thailand and Indonesia have also been actively processing and utilizing rubber wood in furniture making to export world wide (Ratnasingam and Scholz 2008).

In Bangladesh, ages of some rubber tree plantations done by BFIDC were reached to more than 40 years, after which the trees ceased latex production. The trees after clear felling needed replanting the area. To estimate the production of total volume and timber volume for rubber tree before clear felling it is necessary the volume table to estimate volumes. The volume tables of trees are also necessary for economic evaluation, future management, utilization, research purposes and to estimate the quantity of wood during harvest of the tree species (Latif and Islam 2001). But, volume tables for rubber tree, planted are not available in Bangladesh. This bulletin presents the methods and equations derived for volume estimation and stand volume tables preparation for rubber tree and different clones on the basis of equations best suited.

## Materials and Methods

### Measurement of trees

Data were collected from available plantations in established rubber gardens in Bangladesh during January 2011 to June 2011. A total of 583 standing rubber trees representing different girth classes were selected at random for preparation of mathematical volume functions and tables. Out of these 583 trees 195 were standing clone PRIM-600 trees and 388 standing seedling origin rubber trees. Tree girths at breast height

(GBH) in cm and total height in meter were measured with measuring tape and Haga-altimeter respectively. The collected data were categorized on the basis of GBH and height of the trees. The GBH-height class distribution of the sample trees are given in Table 1. The girth and bark thickness at one meter intervals were measured by climbing the trees with a ladder. The bark thicknesses of the samples were measured with a bark gauge.

**Table 1. GBH and total height class distribution of the sampled trees selected for volume estimation of rubber tree, (*Hevea brasiliensis*. Muell Arg.)**

Species	GBH (cm)	Number of tree in the Height (m)					Total
		9	14	19	24	29	
Combined	40	14	40				54
	60	2	70	31	6	1	110
	80	1	44	44	6		95
	100		18	36	21	1	76
	120		12	44	25	12	93
	140		6	43	23	15	87
	160		1	23	19	2	45
	180			4	9		13
	200			2	5		7
	240				2	1	3
	<b>Total</b>	<b>17</b>	<b>191</b>	<b>227</b>	<b>116</b>	<b>32</b>	<b>583</b>
Seedling	40	8	29				37
	60		65	11			76
	80		27	25	2		54
	100		12	19	9		40
	120		10	38	3		51
	140		6	42	16		64
	160		1	23	19		43
	180			4	9		13
	200			2	5		7
	240				2	1	3
	<b>Total</b>	<b>8</b>	<b>150</b>	<b>164</b>	<b>65</b>	<b>1</b>	<b>388</b>
Clone PRIM 600	40	6	11				17
	60	2	5	20	6	1	34
	80	1	17	19	4		41
	100		6	17	12	1	36
	120		2	6	22	12	42
	140			1	7	15	23
	160					2	2
	180						
	200						
	240						
	<b>Total</b>	<b>9</b>	<b>41</b>	<b>63</b>	<b>51</b>	<b>31</b>	<b>195</b>

## Compilation of data

Volumes of all sections except top and bottom section were determined by using the mean cross-sectional areas of the two ends of each section following Smalian's formula cubic volume =  $[(B+b)/2]L$ , where B= the cross-sectional area at the large end of the log, b= the cross-sectional area at the small end of the log, and L= log length (Anon. 2011). In determining the volume of bottom sections, the formulae used for calculating the volume of a cylinder was considered. Assuming the top section as cone the volume was computed to one third of the cylindrical volume of the portion. We considered the top end diameter measurement for each tree as the base diameter of the cone. In computing the under bark volume of the tree the volume of top section i.e. cone was ignored. The volume of the tree is the sum of the volume of total sections found in a tree. The individual tree volumes (V), GBH (G) and total height (H) were variable in regression techniques using various functions and transformations as required in the models.

## Computation of volume function

Multiple regression analysis techniques were used to select the best suited model equations. The following 15 models were tested to select the equation of best fit with different variables as follows.

1.  $V = a + bG$
2.  $V = a + bG + cG^2$
3.  $V = a + bG^2$
4.  $V = bG + cG^2$
5.  $V = a + bG^2H$
6.  $V = a + bG + cH$
7.  $V = a + bG + cG^2H$
8.  $V = a + bG + cGH$
9.  $V = a + bG + cH + dGH$
10.  $V = a + bG + cH + dG^2H$
11.  $V = a + bG^2 + cH + dGH$
12.  $V = a + bG^2 + cH + dG^2H$
13.  $V = a + bG^2 + cGH + dG^2H$
14.  $\log(V) = a + b \log(G)$
15.  $\log(V) = a + b \log(G) + c/\log(H)$

Where: V = total volume over bark in cubic meters,

G = girth at breast height in centimeters,

H = total height in meters,

a = the regression constant and b, c and d are regression coefficients.

The logarithmic functions are to the base e.

Following original and transformed variables were used to select the best suited regression models:

Dependent variables: V, Log (V),

Independent variables: G, G<sup>2</sup>, H, GH, G<sup>2</sup>H, Log (G), Log(H)

The dependent variables mentioned above were regressed with the independent variables.

The equations of the best fit based on the highest multiple coefficients of determination; F-ratio and lowest residual mean square were chosen. Models for estimation of the total volume over bark, total volume under bark and timber volume were selected. The volumes up to 50 cm top end girth may be used as timber for the species. The volumes less up to 50 cm girth may be used for poles, house posts and fuel wood.

### Model Validation

**Statistical validation:** The best suited models were tested with a set of data recollected from 30 trees of different girth classes and complied in the same procedure as earlier. The actual volumes of these trees were collectively compared with the corresponding volume predicted by the selected models. The independent tests for validation were the absolute deviation percent, paired t-test, chi-square test and 45 degree line test (Islam *et al.* 1992). The independent tests for validation criteria were:

(1) The paired t-test :  $t = \frac{\sum(A - E)}{n}$   
 (Dawkins 1975) with n-1 degrees of freedom at 5% level.

(2) Chi-square test :  $\chi^2 = \frac{\sum(A - E)^2}{E}$   
 with n-1 degrees of freedom at 5% level.

(3) Percent Absolute Deviation (%AD)  $\%AD = \frac{\sum(A - E)}{\sum A} \times 100$

Where, A = Actual volume, E = Estimated volume

**Biological principle tests:** The predicted values were plotted against girth at breast height (one way) for different segment. The biological requirement is that the yield curves should be monotonically increasing (Latif and Islam 2001).

### Results and Discussions

The regression model numbers 2, 7, 9, 11, 14 and 15 showed the highest value of coefficients of determination for one way and two way volume equations in combined, seedling origin tree and Clone PRIM 600 rubber trees. The volume equations have been selected for estimation of the total volume over bark (Vob) & total volume under bark (Vub) and timber volume overbark (V<sub>Timob</sub>) & underbark (V<sub>Timub</sub>) from these models. The coefficients of determination for selected volume equations are within 0.83 to 0.98 of different species of rubber trees of different proportion are given table 2. This means that the selected models describe over 83 to 98 percent of the total variations. The best fitted models were selected for estimation of volume on GBH (G) and total height (H). The selected volume equations the clones and seedling origin rubber trees of different proportions are given in Table 2.

Table 2. Volume equation of rubber tree (*Hevea brasiliensis*. Muell Arg.) at different proportion

Species	Volume equations	R <sup>2</sup>	N
Combined	$\ln(V_{ob}) = -10.5628 + 2.1502 \times \ln(G)$	0.95	583
	$\ln(V_{ob}) = -11.2768 + 1.8795 \times \ln(G) + 0.6928 \times \ln(H)$	0.97	583
	$\ln(V_{ub}) = -10.6451 + 2.1607 \times \ln(G)$	0.95	583
	$\ln(V_{ub}) = -11.3509 + 1.8930 \times \ln(G) + 0.6848 \times \log(H)$	0.97	583
	$V_{Timob} = -0.2758 + 0.0036 \times G + 0.000031 \times G^2$	0.86	466
	$V_{Timub} = -0.2598 + 0.0033 \times G + 0.00003 \times G^2$	0.86	466
	$V_{Timob} = 0.0302 + 0.000006 \times G^2 - 0.0288 \times H + 0.00046 \times G \times H$	0.91	466
	$V_{Timub} = 0.02506 + 0.0000064 \times G^2 - 0.02766 \times H + 0.00044 \times G \times H$	0.91	466
Seedling origin tree	$\ln(V_{ob}) = -10.4946 + 2.1365 \times \ln(G)$	0.93	388
	$\ln(V_{ob}) = -11.355075 + 1.90505 \times \ln(G) + 0.67956 \times \ln(H)$	0.96	388
	$\ln(V_{ub}) = -10.58495 + 2.14861 \times \ln(G)$	0.93	388
	$\ln(V_{ub}) = -11.43443 + 1.92013 \times \ln(G) + 0.670876 \times \ln(H)$	0.96	388
	$V_{Timob} = -0.380878 + 0.00564 \times G + 0.000021 \times G^2$	0.83	297
	$V_{Timob} = 0.1795125 - 0.000825 \times G - 0.04131 \times H + 0.00058 \times G \times H$	0.89	297
	$V_{Timub} = -0.36448 + 0.0053378 \times G + 0.0000215 \times G^2$	0.83	297
	$V_{Timub} = 0.180655 - 0.000846 \times G - 0.040676 \times H + 0.00057 \times G \times H$	0.89	297
Clone PRIM 600	$V_{ob} = 0.01097 - 0.00064 \times G + 0.000055 \times G^2$	0.96	195
	$V_{ob} = -0.04833 + 0.00215 \times G + 0.0000019 \times G^2 \times H$	0.97	195
	$V_{ub} = 0.016931 - 0.00085 \times G + 0.000055 \times G^2$	0.96	195
	$V_{ub} = -0.04296 + 0.00195 \times G + 0.0000019 \times G^2 \times H$	0.98	195
	$V_{Timob} = -0.03239 - 0.001618 \times G + 0.000053 \times G^2$	0.96	101
	$V_{Timob} = -0.128147 + 0.00171 \times G + 0.0000017 \times G^2 \times H$	0.96	101
	$V_{Timub} = -0.021049 - 0.00185 \times G + 0.000053 \times G^2$	0.96	101
	$V_{Timub} = -0.11859 + 0.00152 \times G + 0.0000017 \times G^2 \times H$	0.96	101

Where:

G = girth at breast height in centimeter

H = total height in meters

V<sub>ob</sub> = total volume overbark in cubic meters

V<sub>ub</sub> = total volume underbark in cubic meters

V<sub>Timob</sub> = timber volume overbark in cubic meters

V<sub>Timub</sub> = timber volume underbark in cubic meters

### Validation of the selected models

The models developed for volume estimation of rubber tree were verified with the volumes of 30 trees of the clones and seedling origin measured for rubber tree with paired t-test, chi-square test to test the

goodness of fit and 45 degree line test. The computed, t-values and chi-square values are less than tabular values at 5% level of significance. The predicted values tend to make an angle of about 45 deg. This means that there is no significant difference between the observed and the predicted values. The values, chi-square values and slopes are given below:

Table 3 : Different statistics of the selected volume equations/functions

Species	Type of model	t-value	chi-square value	slope (Degree)
Combination	One Way	1.030	1.76264	44
	Two Way	0.738	1.03989	44
Seedling	One Way	-0.548	0.63088	44
	Two Way	-0.474	0.30755	45
PRIM600	OneWay	-0.362	0.34807	43
	Two Way	-0.411	0.31732	43

Therefore, the selected models may be used for rubber tree species within the data range for preparation of the volume tables. After the validation test, volume tables for various proportions of different species were prepared for ready use and are presented in Table 4 to 16. The selected volume models of two segment volume namely total volume over bark, and timber volume over bark of rubber tree of different clones of a seedling origin also satisfied biological criteria of yield curves. The predicted values of total volume over bark and timber volume over bark were plotted against girth at breast height. The growth curves conform with the ideal attributes of biological growth curve. The yield curves are monotonically increasing (Fig. 1) with girth at breast height.

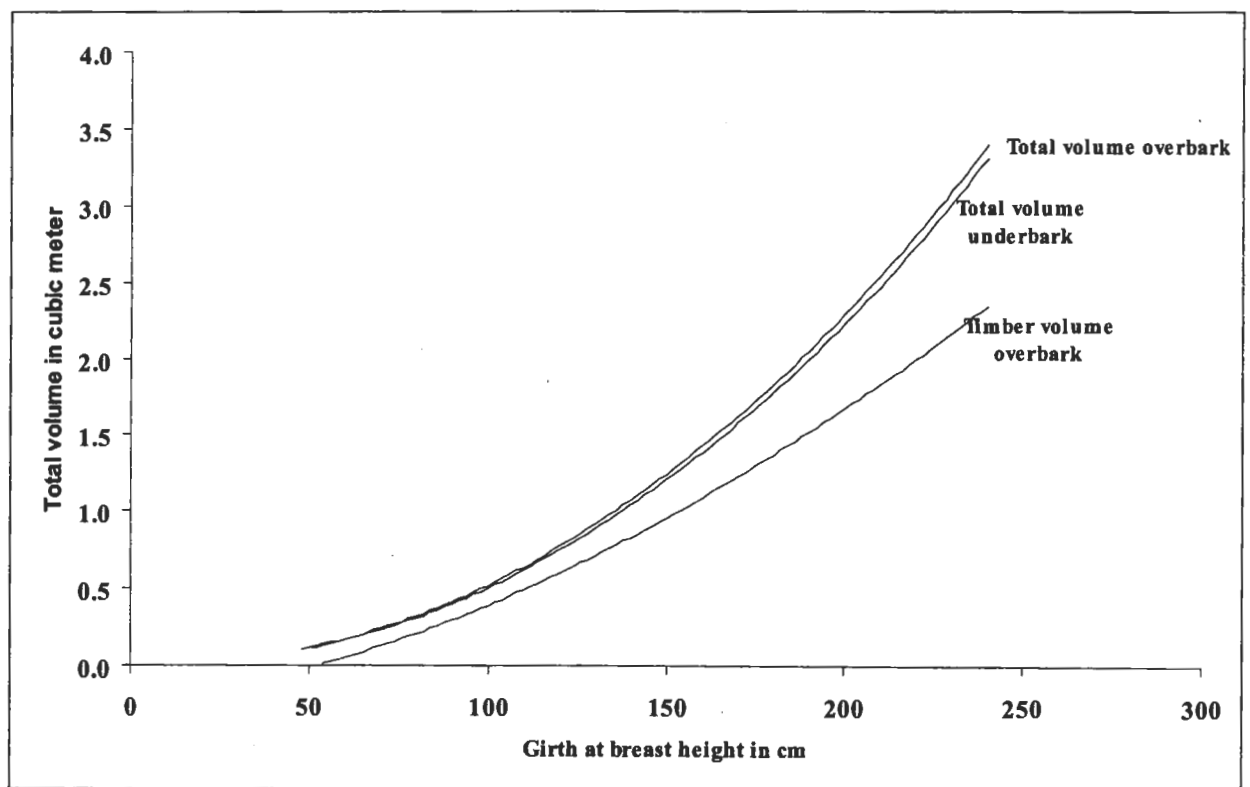


Figure-1: Yield curves of rubber tree (*Hevea brasiliensis*, Muell Arg.) (Combination) in different segments

## GBH-Basal Girth (BG) Relationships

Sometimes the trees are removed without recording the girth at breast height (GBH) and total height. Then, it is not possible to estimate the volumes of the removed trees. But it is necessary to have the estimates of volumes of the removed trees to have an account of the harvest from the plantations. The stump girths can be measured while the stumps are still there after removal of the trees. Therefore, GBH and stump girth relationship (basal girth at about 15 cm above ground level) were established to estimate the GBH of the removed trees first followed by the estimation of the volume of the removed trees. The GBH and stump girth (BG) relationship of the rubber tree (Combination, Seedling origin tree and Clone PRIM-600) are given below:

Combination	$GBH = 8.0464 + 0.6987 \times BG$	$R^2 = 0.93$
Seedling	$GBH = 15.4595 + 0.6554 \times BG$	$R^2 = 0.91$
Clone PRIM600	$GBH = -1.0747 + 0.7594 \times BG$	$R^2 = 0.94$

We estimate volume for ready use and presented in Tables 4-16. The volume equations and tables are applicable for rubber tree (Combination, Seedling and PRIM600) planted in Bangladesh.

### Confidence limit

These volume tables should not be used to estimate volumes of individual trees in a stand. These tables may be used for the mean tree of a stand which may be multiplied by the number of stems to get the total volume of the stand. Estimation of the volumes for the trees outside the height and GBH ranges shown in the stand table should only be done with caution.

### Procedures to use Volume equations and Tables

Take the measurements of girth(s) at breast height (GBH) and total height(s) of the desired tree(s). Then, choose the corresponding volume equations and then substitute these values in predicted equation of the species. For example, let the GBH and height of a selected combined rubber tree are 90 cm and 20 m respectively. Then, the total volume for this tree is:

$$\begin{aligned} \ln(V_{ob}) &= -11.2768 + 1.8795 \times \ln(G) + 0.6928 \times \ln(H) \\ &= -11.2768 + 1.8795 \times \ln(90) + 0.6928 \times \ln(20) \\ &= -0.7442 \\ V &= \text{Exp.}(\ln(V)) = 0.475 \text{ m}^3 \end{aligned}$$

If the measured GBH and total height coincide with the tabular GBH and total height then the tabular values may only be used directly. The one way volume table (GBH-volume tables and equations) may similarly be used. The girths have to divide by the factor 2.54 to get inches from centimeters. The height should be multiplied with 3.281 to convert meter into feet. Similarly, volume should be multiplied by the factor 35.32 to get cubic feet from cubic meters.



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**Table 4. One way volume table in cubic meter for Rubber tree growing in Bangladesh.**

Girth breast height (cm)	Combined (Clone PRIM 600 and Seedling origin tree)				Seedling origin tree Volume				Clone PRIM-600 trees			
	Total Volume (m <sup>3</sup> )		Timber Volume (m <sup>3</sup> )		Total Volume (m <sup>3</sup> )		Timber Volume (m <sup>3</sup> )		Total Volume (m <sup>3</sup> )		Timber Volume (m <sup>3</sup> )	
	Over bark	Under bark	Over bark	Under bark	Over bark	Under bark	Over bark	Under bark	Over bark	Under bark	Over bark	Under bark
30	0.0388	0.0370			0.0396	0.0377			0.0418	0.0413		
32	0.0446	0.0426			0.0455	0.0434			0.0474	0.0465		
34	0.0508	0.0485			0.0518	0.0494			0.0535	0.0521		
36	0.0574	0.0549			0.0585	0.0558			0.0600	0.0582		
38	0.0645	0.0617			0.0657	0.0627			0.0669	0.0647		
40	0.0720	0.0689			0.0733	0.0700			0.0743	0.0716		
42	0.0800	0.0766			0.0813	0.0778			0.0821	0.0790		
44	0.0884	0.0847			0.0898	0.0859			0.0904	0.0868		
46	0.0973	0.0932			0.0988	0.0945			0.0991	0.0951		
48	0.1066	0.1022			0.1082	0.1036			0.1083	0.1038		
50	0.1164	0.1116			0.1181	0.1131			0.1179	0.1130	0.0196	0.0195
52	0.1266	0.1215			0.1284	0.1230			0.1280	0.1226	0.0273	0.0267
54	0.1373	0.1318	0.0069	0.0070	0.1392	0.1334			0.1385	0.1326	0.0353	0.0343
56	0.1485	0.1426	0.0208	0.0203	0.1504	0.1443			0.1494	0.1431	0.0438	0.0423
58	0.1601	0.1538	0.0350	0.0339	0.1621	0.1556	0.0184	0.0176	0.1608	0.1540	0.0526	0.0507
60	0.1722	0.1655	0.0494	0.0477	0.1743	0.1673	0.0347	0.0333	0.1726	0.1654	0.0620	0.0596
62	0.1848	0.1777	0.0640	0.0618	0.1869	0.1795	0.0512	0.0492	0.1849	0.1772	0.0717	0.0688
64	0.1979	0.1903	0.0789	0.0761	0.2001	0.1922	0.0679	0.0654	0.1976	0.1895	0.0819	0.0786
66	0.2114	0.2034	0.0940	0.0906	0.2137	0.2054	0.0848	0.0816	0.2108	0.2022	0.0924	0.0887
68	0.2254	0.2169	0.1094	0.1054	0.2277	0.2190	0.1018	0.0981	0.2244	0.2153	0.1035	0.0993
70	0.2399	0.2310	0.1251	0.1205	0.2423	0.2330	0.1190	0.1147	0.2384	0.2289	0.1149	0.1102
72	0.2549	0.2455	0.1409	0.1358	0.2573	0.2476	0.1364	0.1315	0.2529	0.2430	0.1268	0.1217
74	0.2703	0.2604	0.1570	0.1514	0.2728	0.2626	0.1539	0.1484	0.2678	0.2574	0.1391	0.1335
76	0.2863	0.2759	0.1734	0.1672	0.2888	0.2781	0.1716	0.1656	0.2832	0.2723	0.1518	0.1458
78	0.3027	0.2918	0.1900	0.1832	0.3053	0.2940	0.1895	0.1829	0.2991	0.2877	0.1649	0.1584
80	0.3197	0.3082	0.2069	0.1995	0.3223	0.3105	0.2076	0.2004	0.3153	0.3035	0.1785	0.1716
82	0.3371	0.3251	0.2240	0.2160	0.3397	0.3274	0.2258	0.2180	0.3320	0.3198	0.1925	0.1851
84	0.3550	0.3425	0.2413	0.2328	0.3577	0.3448	0.2442	0.2359	0.3492	0.3364	0.2069	0.1991
86	0.3735	0.3603	0.2589	0.2498	0.3761	0.3627	0.2628	0.2539	0.3668	0.3536	0.2217	0.2135
88	0.3924	0.3787	0.2767	0.2671	0.3950	0.3810	0.2815	0.2720	0.3848	0.3712	0.2370	0.2283
90	0.4118	0.3975	0.2948	0.2846	0.4145	0.3999	0.3004	0.2904	0.4033	0.3892	0.2527	0.2435
92	0.4317	0.4169	0.3131	0.3024	0.4344	0.4192	0.3195	0.3089	0.4223	0.4076	0.2688	0.2592
94	0.4522	0.4367	0.3317	0.3204	0.4548	0.4390	0.3388	0.3276	0.4417	0.4265	0.2854	0.2753
96	0.4731	0.4570	0.3505	0.3387	0.4757	0.4594	0.3582	0.3464	0.4615	0.4459	0.3023	0.2918
98	0.4946	0.4778	0.3696	0.3572	0.4972	0.4802	0.3778	0.3655	0.4817	0.4657	0.3197	0.3088
100	0.5165	0.4992	0.3889	0.3759	0.5191	0.5015	0.3976	0.3847	0.5025	0.4859	0.3376	0.3261
102	0.5390	0.5210	0.4085	0.3949	0.5415	0.5233	0.4175	0.4040	0.5236	0.5066	0.3558	0.3439
104	0.5620	0.5433	0.4283	0.4142	0.5645	0.5456	0.4376	0.4236	0.5452	0.5277	0.3745	0.3622
106	0.5855	0.5661	0.4483	0.4337	0.5879	0.5684	0.4579	0.4433	0.5673	0.5493	0.3936	0.3808
108	0.6095	0.5895	0.4686	0.4534	0.6119	0.5916	0.4784	0.4632	0.5898	0.5713	0.4131	0.3999
110	0.6340	0.6133	0.4891	0.4734	0.6363	0.6154	0.4990	0.4833	0.6127	0.5937	0.4330	0.4194
112	0.6591	0.6377	0.5099	0.4936	0.6613	0.6397	0.5198	0.5035	0.6361	0.6166	0.4534	0.4393